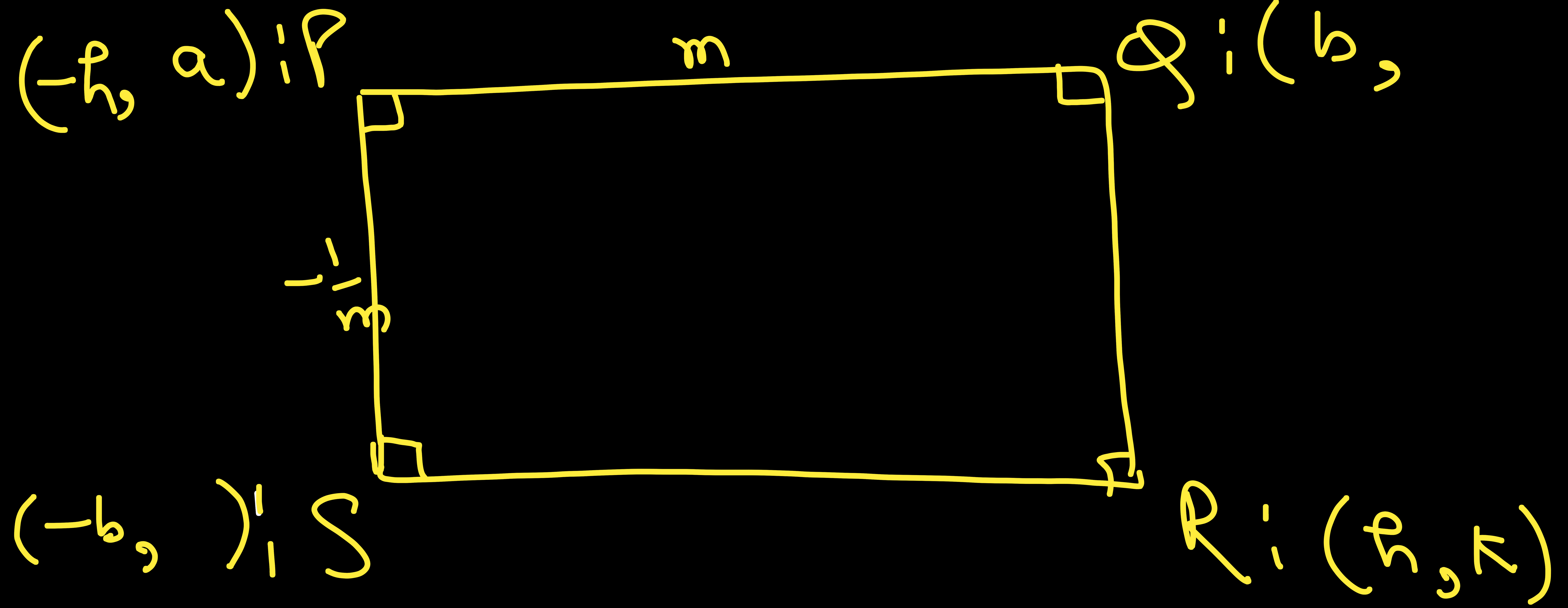
Que

A rectangle PQRS has its side PQ parallel to the line $y = mx$ and vertices P, Q and S on the lines $y = a$, $x = b$ and $x = -b$ respectively. Find the locus of vertex R.

Sol



$$\frac{P+R}{2} = \frac{Q+S}{2}$$

⇓ x-coordinates

$$x_P + h = b - b$$

$$\Rightarrow x_P = -h$$

$$m_{PQ} = m = \frac{y_Q - a}{b+h} \Rightarrow y_Q = m(b+h) + a$$

$$\hookrightarrow m_{QR} = \boxed{-\frac{1}{m} = \frac{m(b+h)+a-k}{b-h}}$$

$(h,k) \to (x,y)$ for locus

$$\boxed{x - b = m^2(b+x) + m(a-y)}$$

R. Locus Ans

Q The true set of real values of a such that the point $M(a, \sin a)$ lies inside the triangle formed by the lines $x-2y+2=0$, $x+y=0$ and $x-y-\pi=0$, is

Sol $M: (a, \sin(a))$

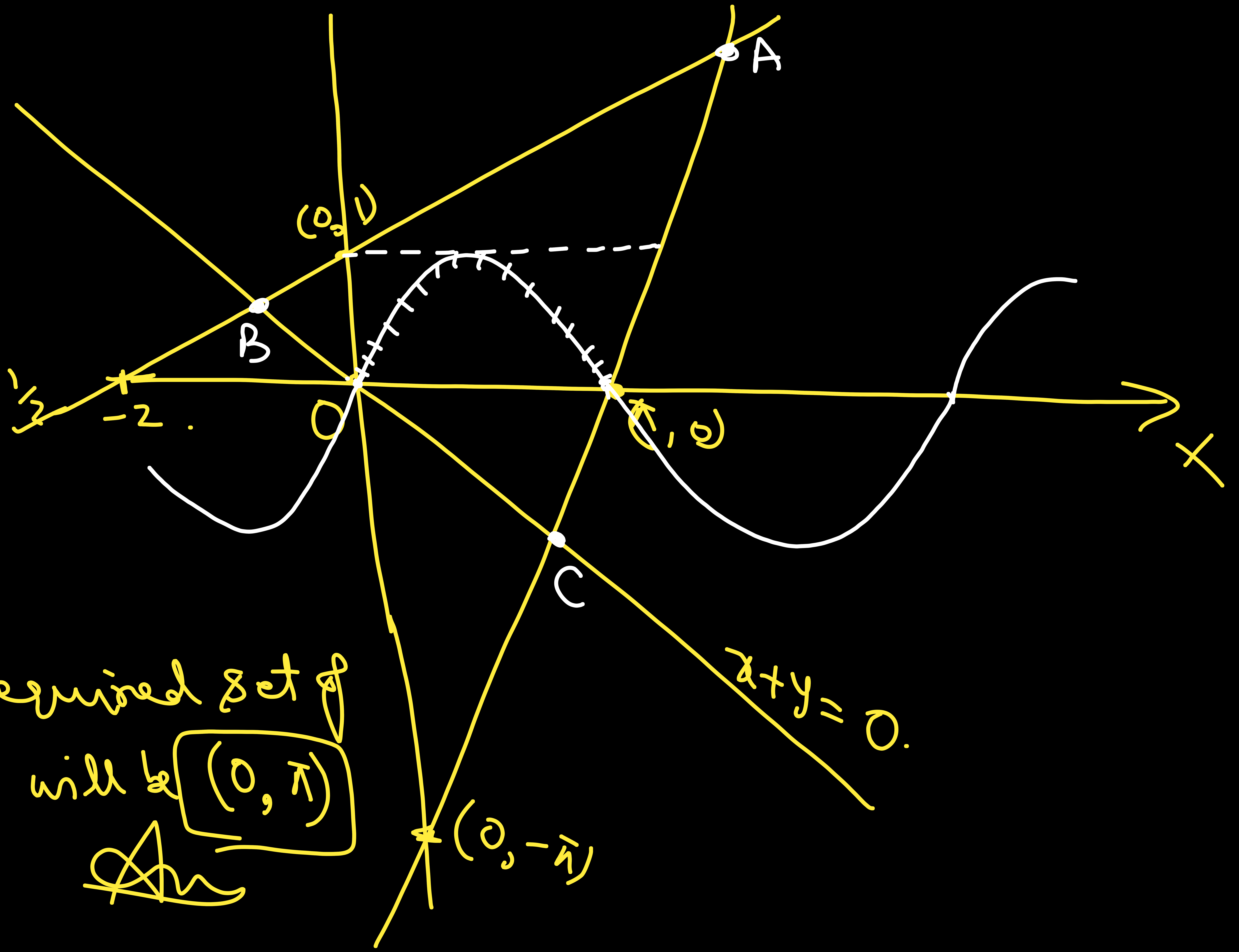
$y = \sin(x)$

Inside the Δ

$x-2y+2=0$

$x+y=0$

$x-y=\pi$



from graph Required set of values of a will be $\boxed{(0, \pi)}$

Ans

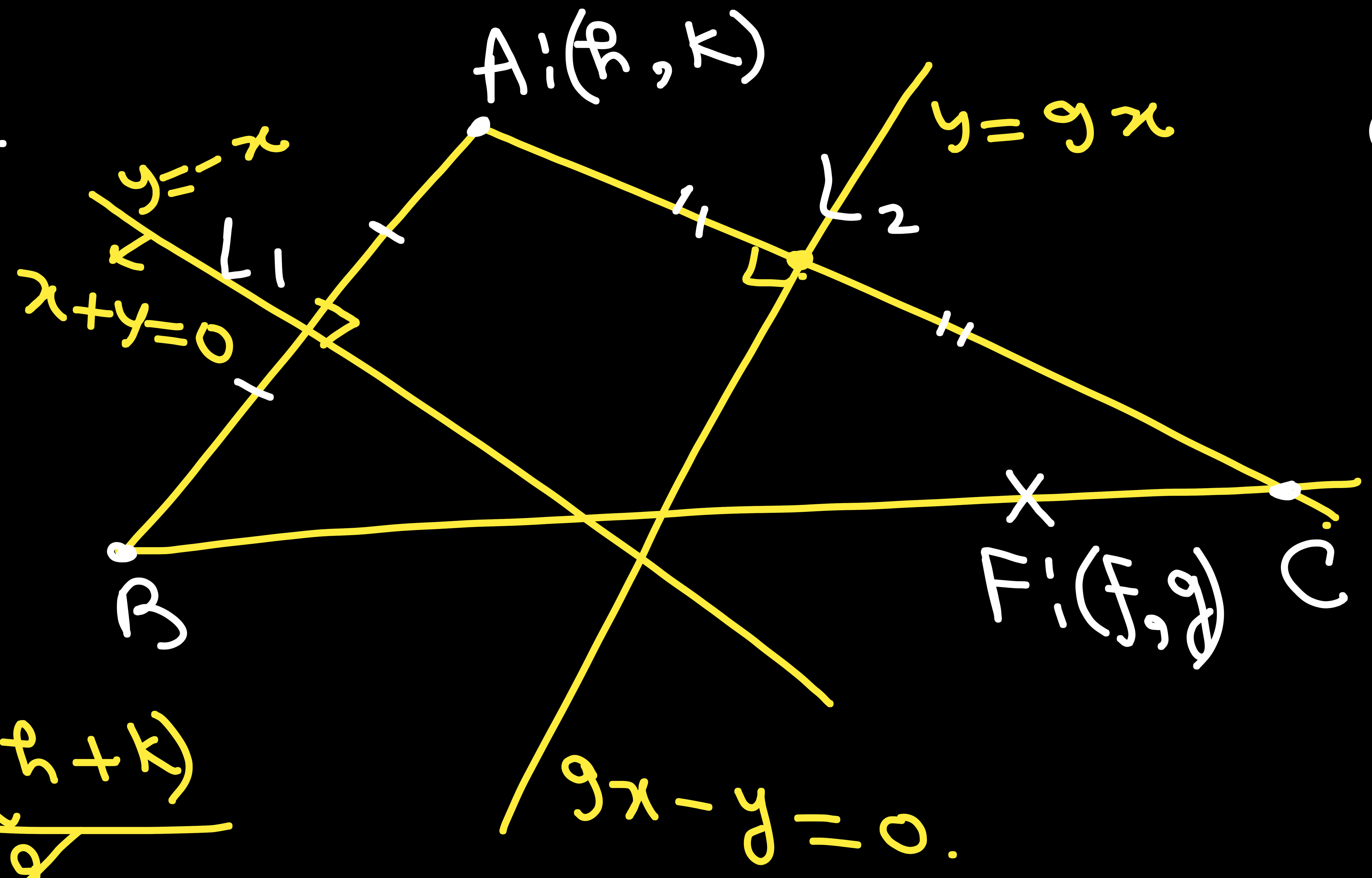
The base of a triangle passes through a fixed point (f, g) and its sides are respectively bisected at right angles by the lines $y^2 - 8xy - 9x^2 = 0$. Determine the locus of its vertex.

Sol

$y^2 - 8xy - 9x^2 = 0.$

$y = 9x$

$y = -x.$



Clearly, reflxn of A in L_1 & L_2 respectively will be B & C.

$B \equiv \frac{x-h}{1} = \frac{y-k}{1} = -\frac{\cancel{2}(h+k)}{\cancel{2}}$

$\Rightarrow B : (-k, -h)$

$C \equiv \frac{x-h}{9} = \frac{y-k}{-1} = -\frac{\cancel{2}(9h-k)}{\cancel{82}\,41} \Rightarrow x = -\frac{9}{41}(9h-k) + h = \frac{-40h + 9k}{41}$

$y = k + \frac{9h-k}{41} = \frac{9h - 40k}{41}$

$B : (-k, -h)$, $\quad C = \left(\dfrac{-40h+9k}{41}, \dfrac{9h-40k}{41} \right)$

as B, f, C are collinear hence

$$\begin{vmatrix} -k & -h & 1 \\ \dfrac{-40h+9k}{41} & \dfrac{9h-40k}{41} & 1 \\ f & g & 1 \end{vmatrix} = 0.$$

for locus

$(h, k) \rightarrow (x, y)$

Que

The vertices of a triangle are $(1, \sqrt{3})$, $(2\cos\theta, 2\sin\theta)$ and $(2\sin\theta, -2\cos\theta)$ where $\theta \in R$. The locus of orthocentre of the triangle is

(A) $(x-1)^2 + (y-\sqrt{3})^2 = 4$ (B) $(x-2)^2 + (y-\sqrt{3})^2 = 4$

(C) $(x-1)^2 + (y-\sqrt{3})^2 = 8$ (D) $(x-2)^2 + (y-\sqrt{3})^2 = 8$

Sol

$A : (1, \sqrt{3}) \Rightarrow OA = 2$

$B : (2\cos(\theta), 2\sin(\theta)) \Rightarrow OB = 2$

$C : (2\sin(\theta), -2\cos(\theta)) \Rightarrow OC = 2.$

$\Rightarrow$ Origin is circumcentre

$(h, k) : H$ — $2 : 1$ — G — C.C. : $(0, 0)$

$$x_G = \frac{h}{\cancel{3}} = \frac{1 + 2\cos(\theta) + 2\sin(\theta)}{\cancel{3}}$$

$$y_G = \frac{k}{\cancel{3}} = \frac{\sqrt{3} + 2\sin(\theta) - 2\cos(\theta)}{\cancel{3}}$$

$$\begin{cases} h-1 = 2\left(\cos(\theta) + \sin(\theta)\right) \\ k-\sqrt{3} = 2\left(\sin(\theta) - \cos(\theta)\right) \end{cases}$$

$$(\ )^2 + (\ )^2 \quad (h-1)^2 + (k-\sqrt{3})^2 = 4\cdot\left(\underbrace{c^2+s^2}_{=1} + \cancel{2sc} + \underbrace{s^2+c^2}_{=1} - \cancel{2sc}\right)$$

$$= 8$$

$$\boxed{(x-1)^2 + (y-\sqrt{3})^2 = 8}$$

A

A C

One

Triangle formed by the lines $x+y=0$, $x-y=0$ and $lx+my=1$. If l and m vary subject to the condition $l^2+m^2=1$, then the locus of its circumcentre is

(A) $(x^2-y^2)^2=x^2+y^2$
(B) $(x^2+y^2)^2=(x^2-y^2)$
(C) $(x^2+y^2)=4x^2y^2$
(D) $(x^2-y^2)^2=(x^2+y^2)^2$

$L_1 \equiv x+y=0$

$L_2 \equiv x-y=0.$

Variable line $\Leftarrow L_3 \equiv lx+my=1$

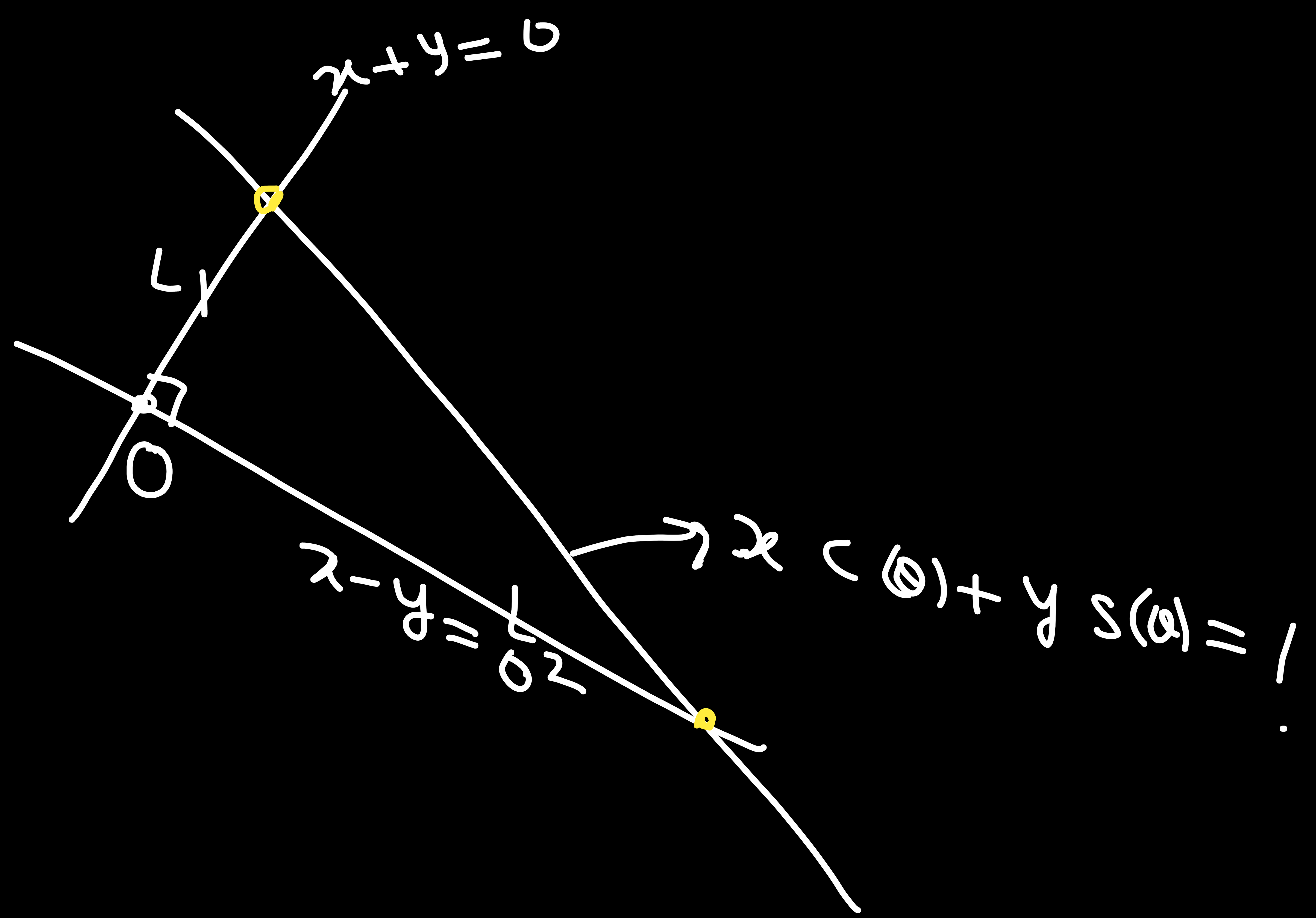
$l^2+m^2=1$

$l=\cos(\theta)$
$m=\sin(\theta)$

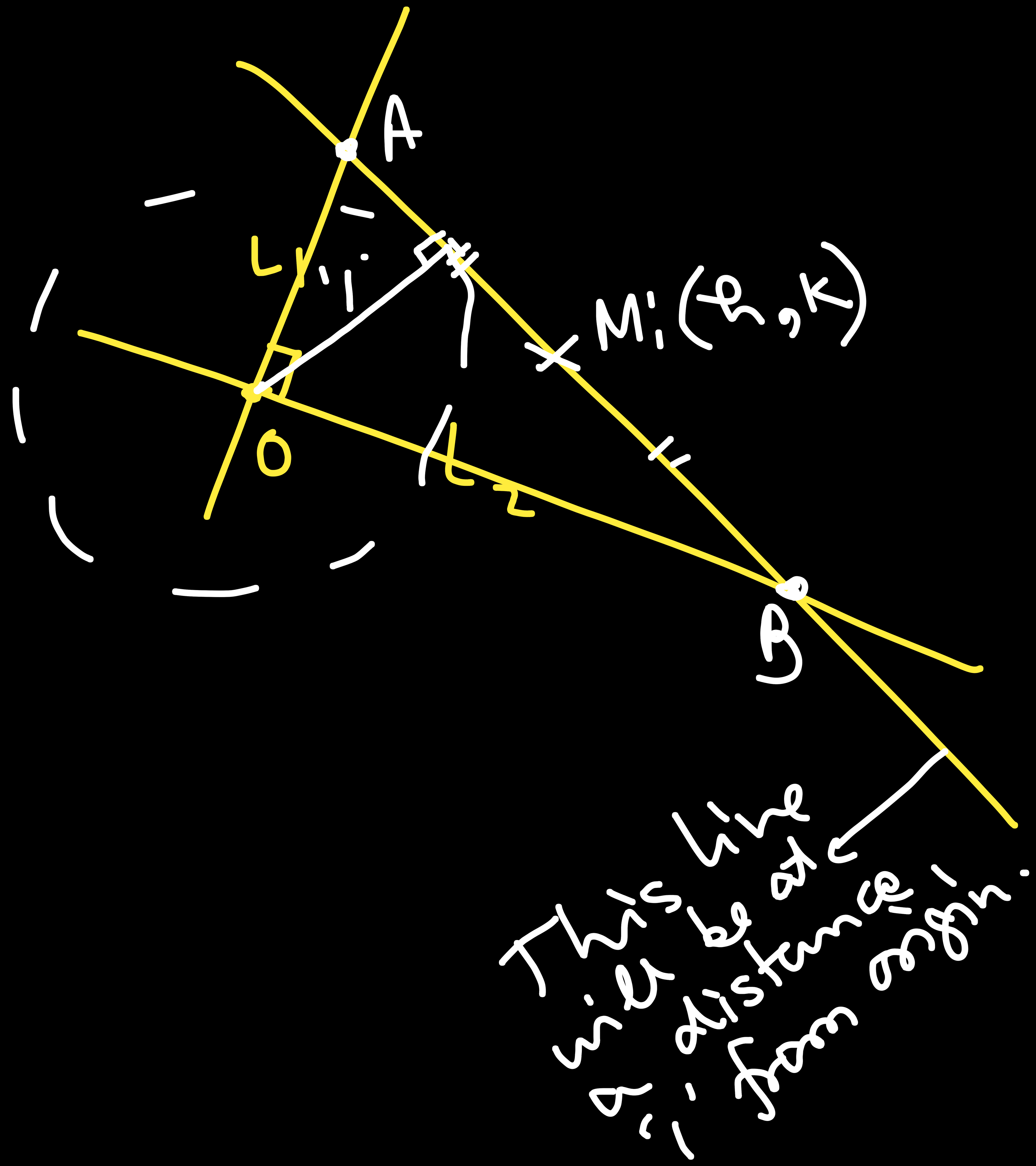
$x\cos(\theta)+y\sin(\theta)=1$

$\Downarrow$

Eqn of tangent to the circle $x^2+y^2=1$



joint Eqn of L_1 & $L_2 \equiv x^2 - y^2 = 0 \longrightarrow$ Conic. (1)



$AB \equiv T = S_1$ (for conic (1) w.r.t. M)

$xh - yk = h^2 - k^2$

$\Downarrow$

$$1 = \frac{|0 - 0 - (h^2 - k^2)|}{\sqrt{h^2 + k^2}}$$

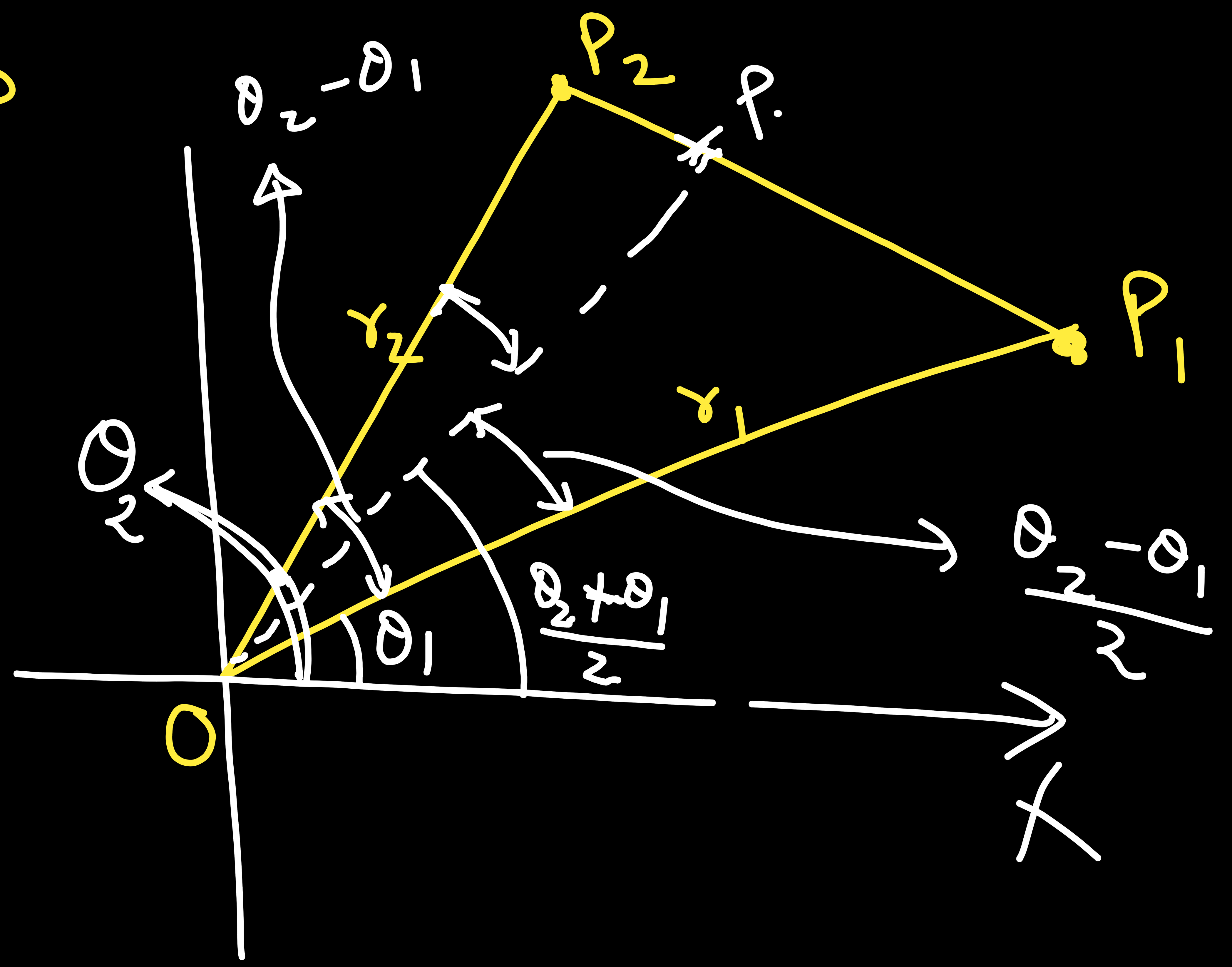
$$\boxed{x^2 + y^2 = (x^2 - y^2)^2}$$

A

Que

Two points P_1 and P_2 are at distances r_1 and r_2 respectively from the origin O and OP_1 and OP_2 make angles θ_1 and θ_2 respectively with the x-axis. Let there be a point P on P_1P_2 such that OP makes an angle $\frac{\theta_2+\theta_1}{2}$ with the x-axis. Show that $OP = \frac{2r_1r_2}{r_1+r_2}\cos\left(\frac{\theta_2 \sim \theta_1}{2}\right)$.

Sol



given $\angle POX = \frac{\theta_1+\theta_2}{2}$.

$\Rightarrow \angle POP_1 = \frac{\theta_2-\theta_1}{2}$

& $\angle POP_2 = \frac{\theta_2-\theta_1}{2}$.

$\Rightarrow$ OP is angular bisector of ΔP_1OP_2.

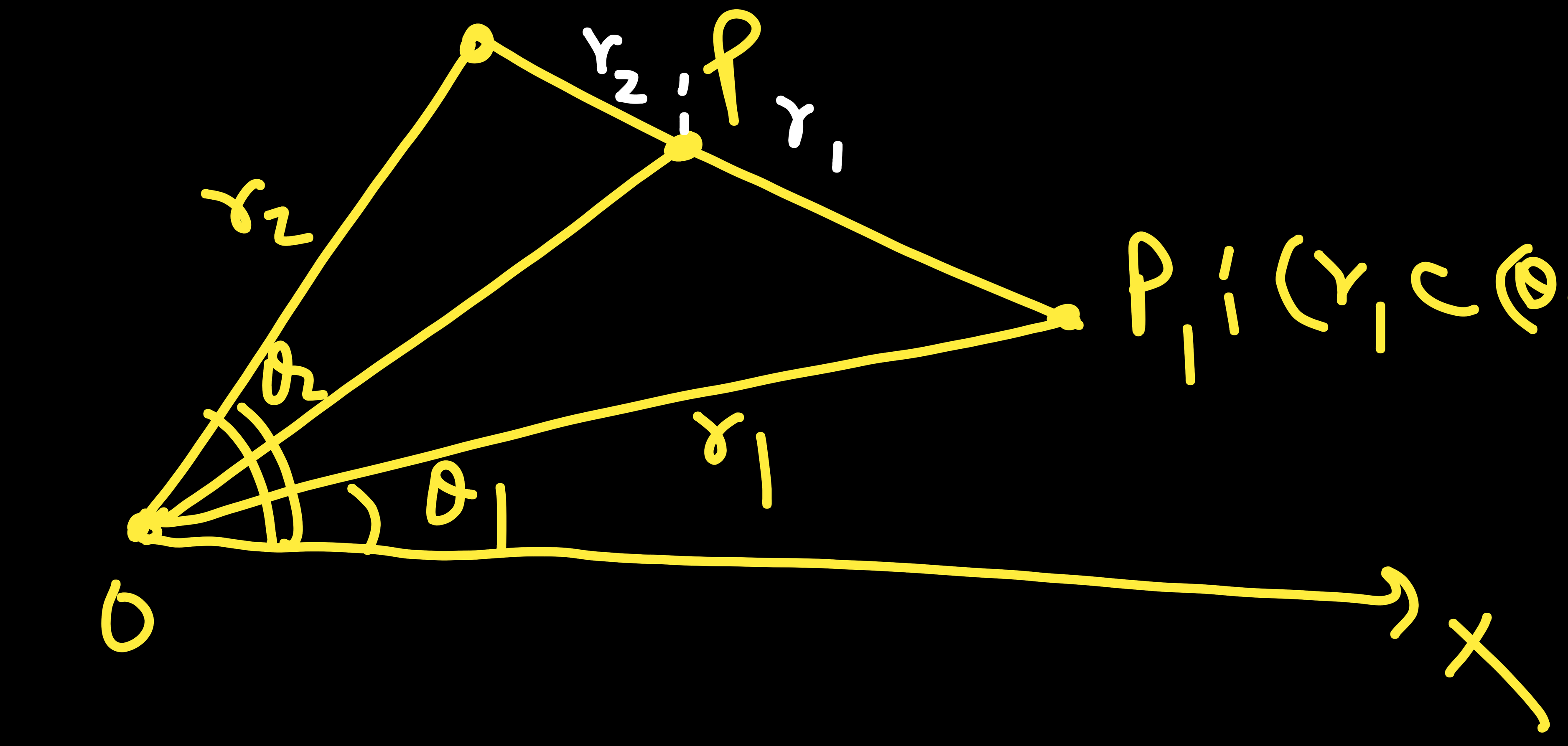
M-1 Length of angular bisector through vertex A

$$= \frac{2bc}{b+c} \cdot \cos\left(\frac{A}{2}\right) = \frac{2r_1 r_2}{r_1+r_2} \cos\left(\frac{\theta_2-\theta_1}{2}\right)$$

H.P.

M-2

$P_2 : (r_2 c(\theta_2), r_2 s(\theta_2))$



$P_1 : (r_1 c(\theta_1), r_1 s(\theta_1))$

$$P \equiv \left(\frac{r_2 r_1 c(\theta_1) + r_1 r_2 c(\theta_2)}{r_1+r_2}, \frac{r_2 r_1 s(\theta_1) + r_1 r_2 s(\theta_2)}{r_1+r_2}\right)$$

$$\equiv \left(\frac{r_1 r_2}{r_1+r_2} 2c\left(\frac{\theta_1+\theta_2}{2}\right) c\left(\frac{\theta_1-\theta_2}{2}\right), \frac{r_2 r_1}{r_1+r_2} . 2s\left(\frac{\theta_1+\theta_2}{2}\right) c\left(\frac{\theta_1-\theta_2}{2}\right)\right)$$

$$OP = \sqrt{x_p^2 + y_p^2} = \frac{2r_1 r_2}{r_1+r_2} \cos\left(\frac{\theta_1-\theta_2}{2}\right) . \sqrt{\cancelto{1}{c^2\left(\frac{\theta_1+\theta_2}{2}\right) + s^2\left(\frac{\theta_1+\theta_2}{2}\right)}}$$

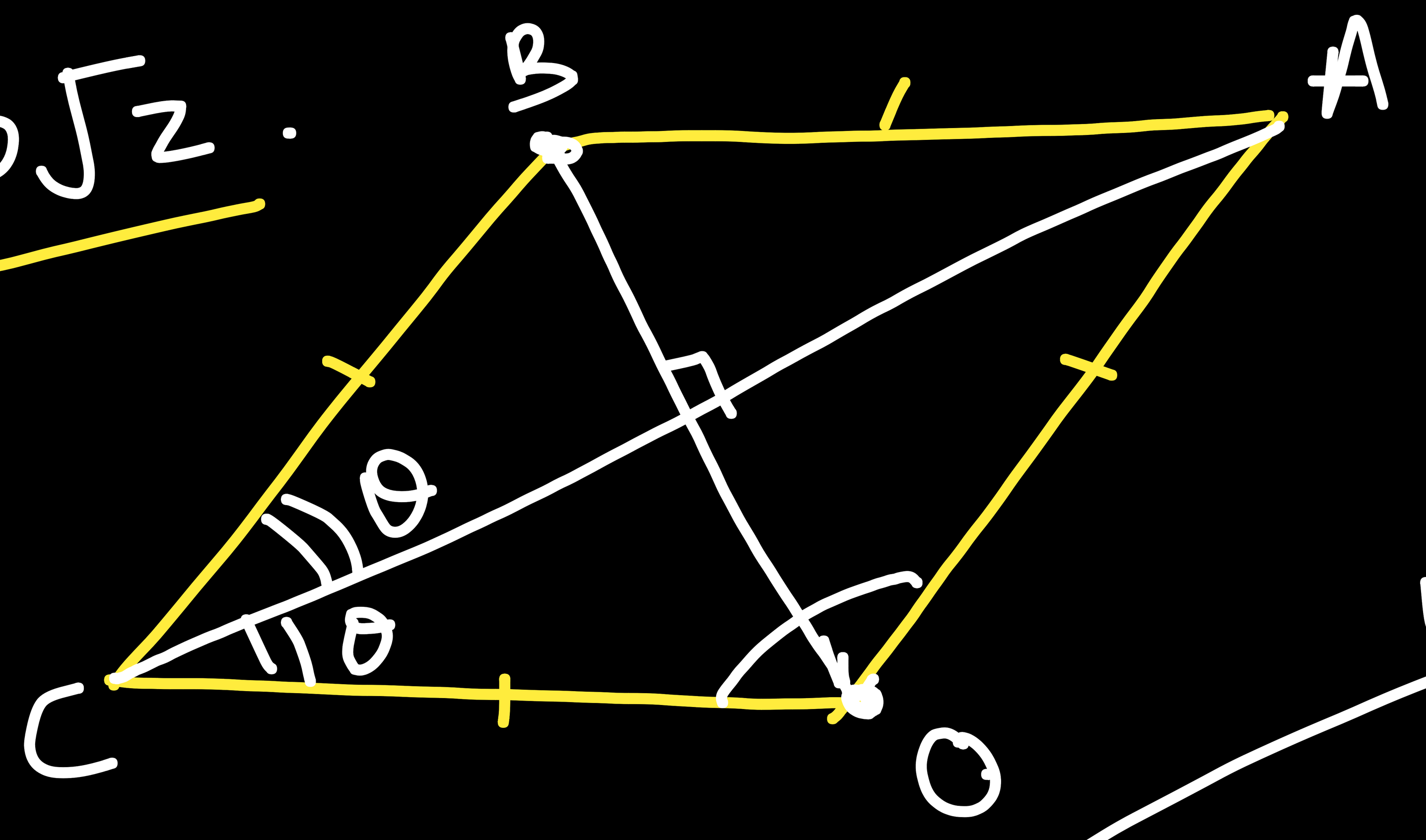
H.P.

One

Two adjacent sides of a rhombus are given by $2x^2 - 47xy + 2y^2 = 0$. If length of the shorter diagonal passing through their point of intersection is $10\sqrt{2}$ then find the possible positions of the vertex lying on it other than origin.

Sol

$OB = 10\sqrt{2}$.



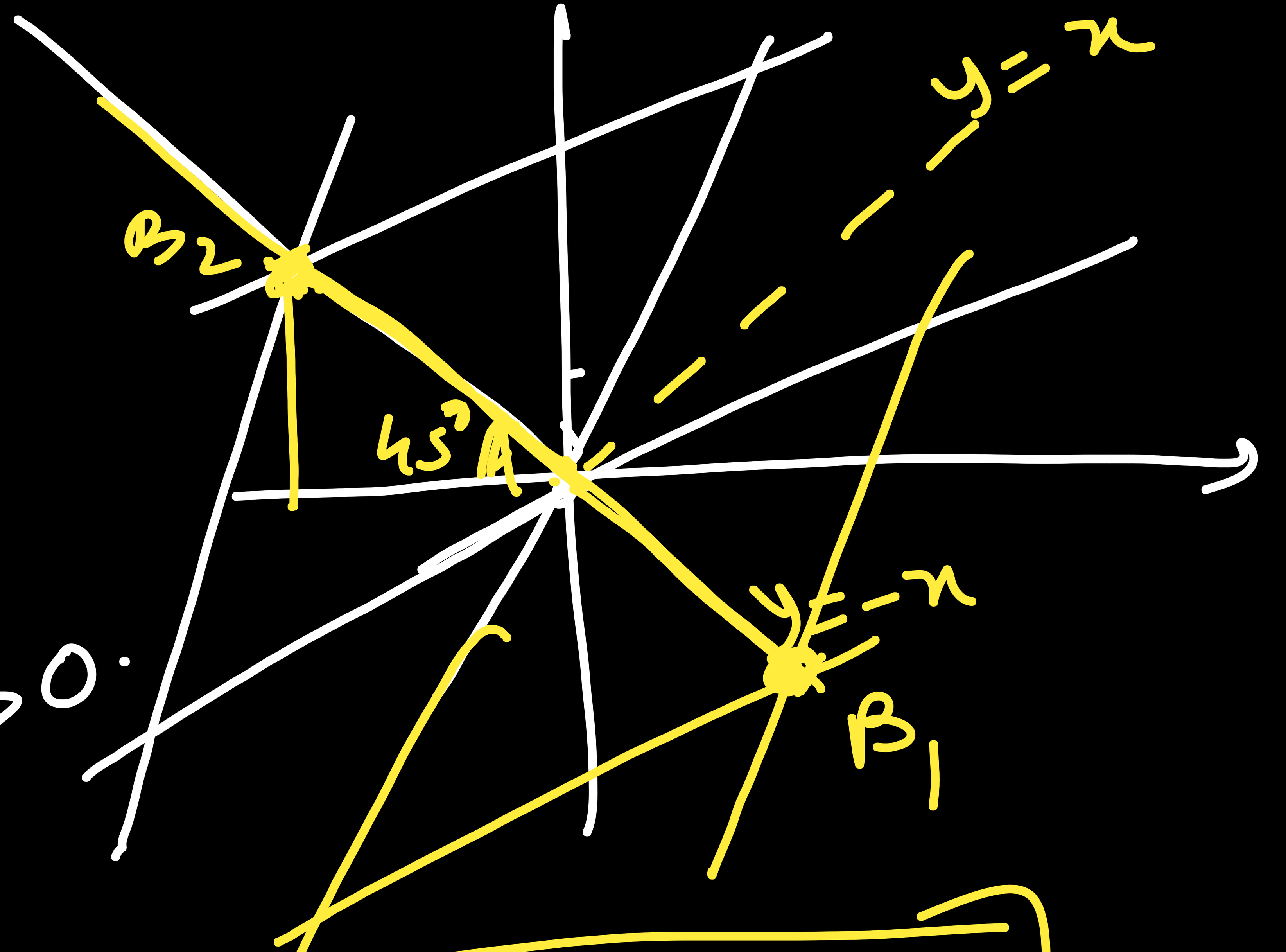
$m_1 m_2$

$m_1 + m_2 = \frac{47}{2} > 0$

$m_1 m_2 = 1 > 0.$

$\Downarrow$

$m_1, m_2 > 0.$



$2x^2 - 47xy + 2y^2 = 0 \longrightarrow a = 2,\ b = 2,\ h = -\frac{47}{2}.$

Angular bisector

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

$$\Rightarrow x^2 - y^2 - \left(\frac{a-b}{h}\right)xy = 0.$$

$$x^2 - y^2 = 0 \Rightarrow x = y \text{ or } x = -y$$

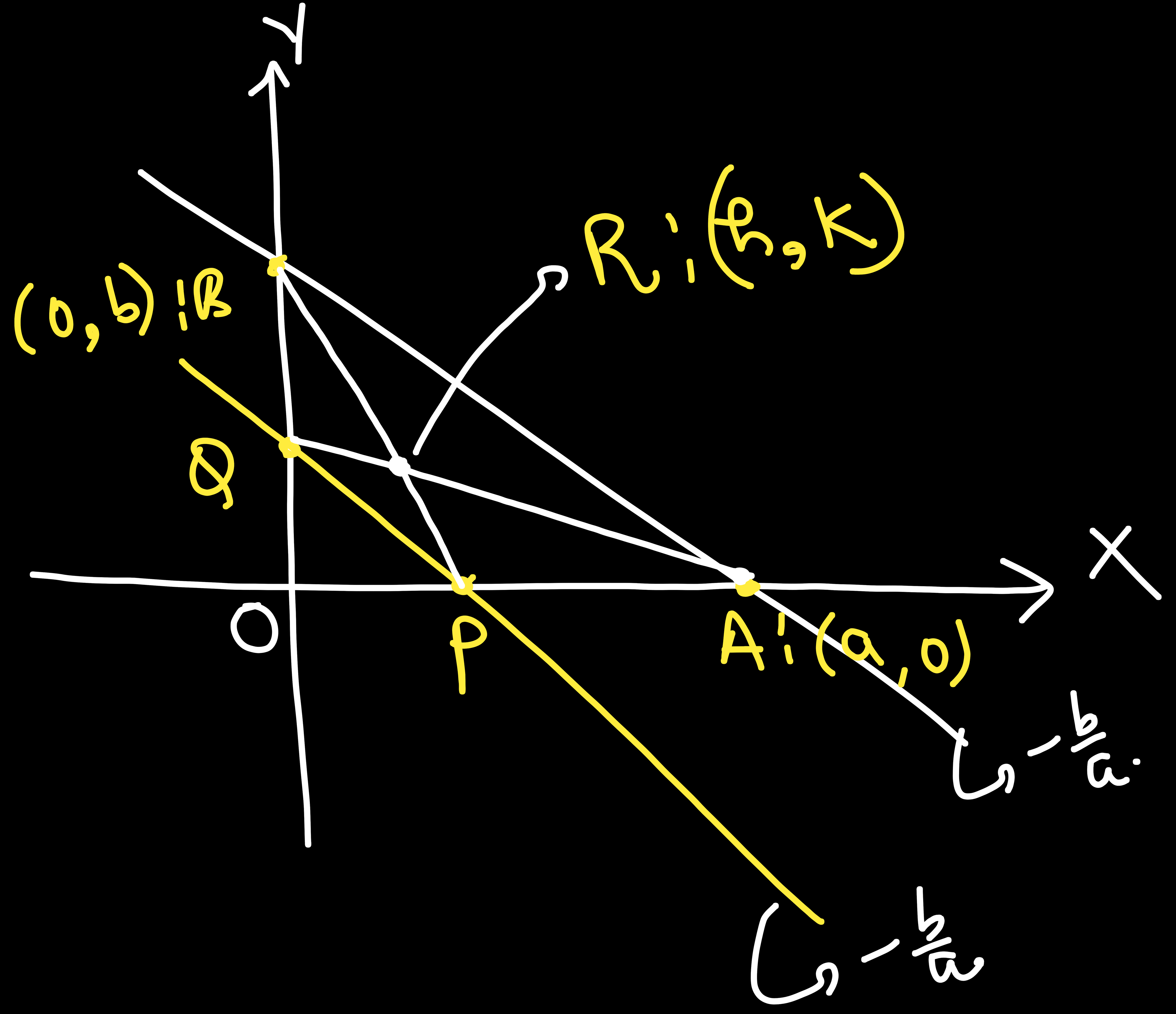
$B_1 : (10, -10)$

or $B_2 : (-10, 10)$

Que

Let the line $\frac{x}{a}+\frac{y}{b}=1$ cuts the x and y axes at A and B respectively. Now a line parallel to the given line cuts the coordinate axis at P and Q and points P and Q are joined to B and A respectively. Find the locus of intersection of the joining lines.

Sol



$PB \equiv y-b=\frac{k-b}{h-0}(x-0)$

$PB \equiv y-b=\left(\frac{k-b}{h}\right)(x) \xrightarrow[y=0]{\text{for P put}} P:\left(\frac{-bh}{K-b},0\right)$

$AQ \equiv y-0=\frac{k-0}{h-a}(x-a)$

$y=\left(\frac{k}{h-a}\right)(x-a)$

for Q put $x=0$ $\rightarrow$ $Q:\left(0,\ \frac{-aK}{h-a}\right)$

Now $m_{PQ}=-\frac{b}{a}=\frac{-\left(\frac{aK}{h-a}\right)-0}{0+\left(\frac{bh}{K-b}\right)} \Rightarrow \frac{b}{a}\cdot\frac{bh}{K-b}=\frac{aK}{h-a}$

$$b^2 h(h-a) = a^2 k(k-b)$$

$$\Downarrow$$

$$\boxed{b^2 x(x-a) = a^2 y(y-b)}$$

Ⓐ $\frac{x}{a} - \frac{y}{b} = 0$

Ⓑ $\frac{x}{a} + \frac{y}{b} = 0$

Ⓒ $\frac{x}{b} + \frac{y}{a} = 0$

Ⓓ None of these

$$\Rightarrow b^2x^2 - a^2y^2 - ab^2x + ba^2y = 0.$$

$$(bx-ay)(bx+ay) - ab(bx-ay) = 0.$$

$$(bx-ay)(bx+ay-ab) = 0.$$

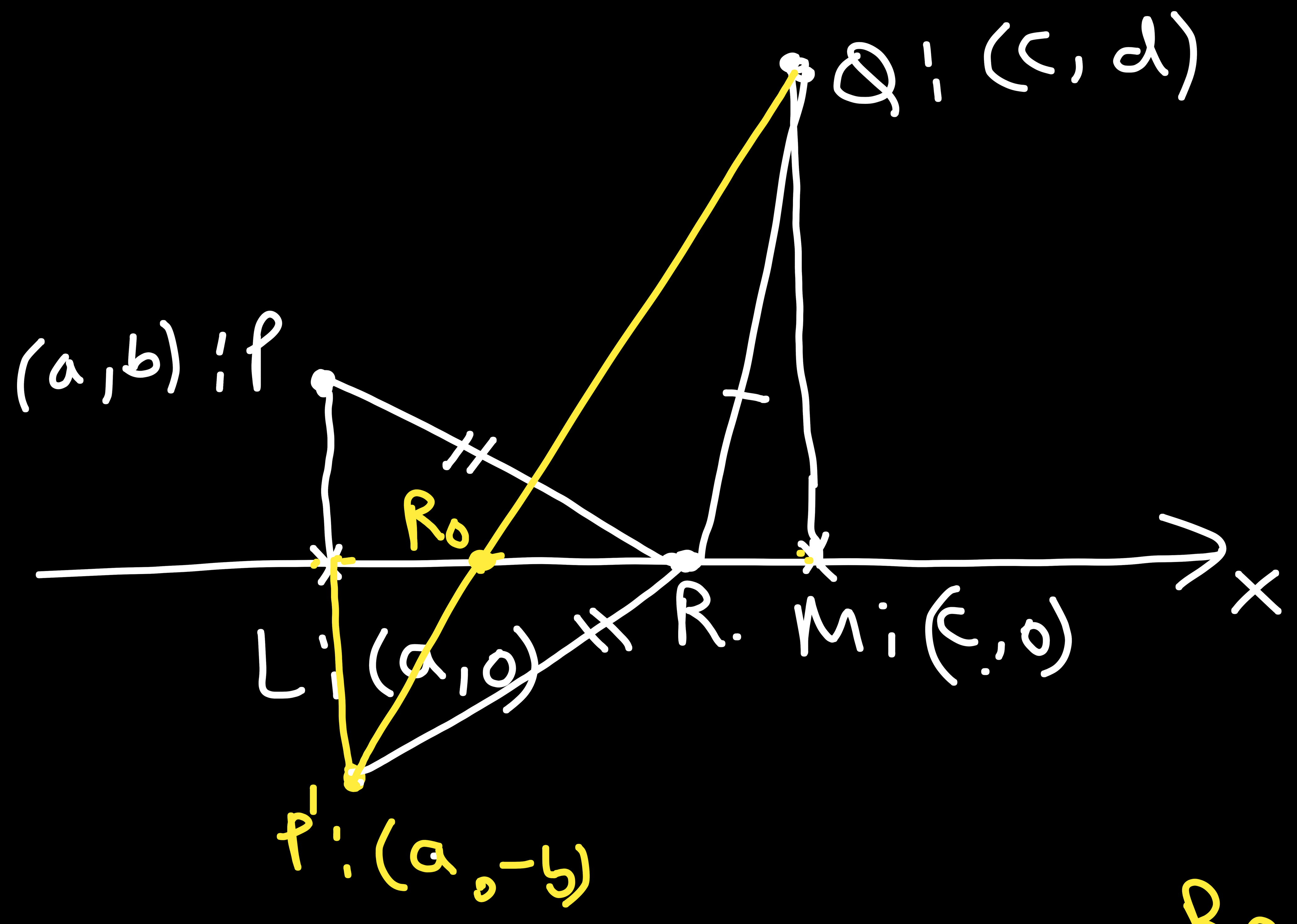
$$bx - ay = 0 \Rightarrow \boxed{\frac{x}{a} - \frac{y}{b} = 0}$$

$$bx+ay-ab = 0 \Rightarrow \frac{x}{a} + \frac{y}{b} = 1$$

Que

Let P ≡ (a, b), Q ≡ (c, d) and 0<a<b<c<d, L ≡ (a, 0), M ≡ (c, 0), R lies on x – axis such that PR + RQ is minimum, then find LR : RM.

Sol



To minimize PR + RQ we can minimize P'R + RQ which will be minimum for R_0.

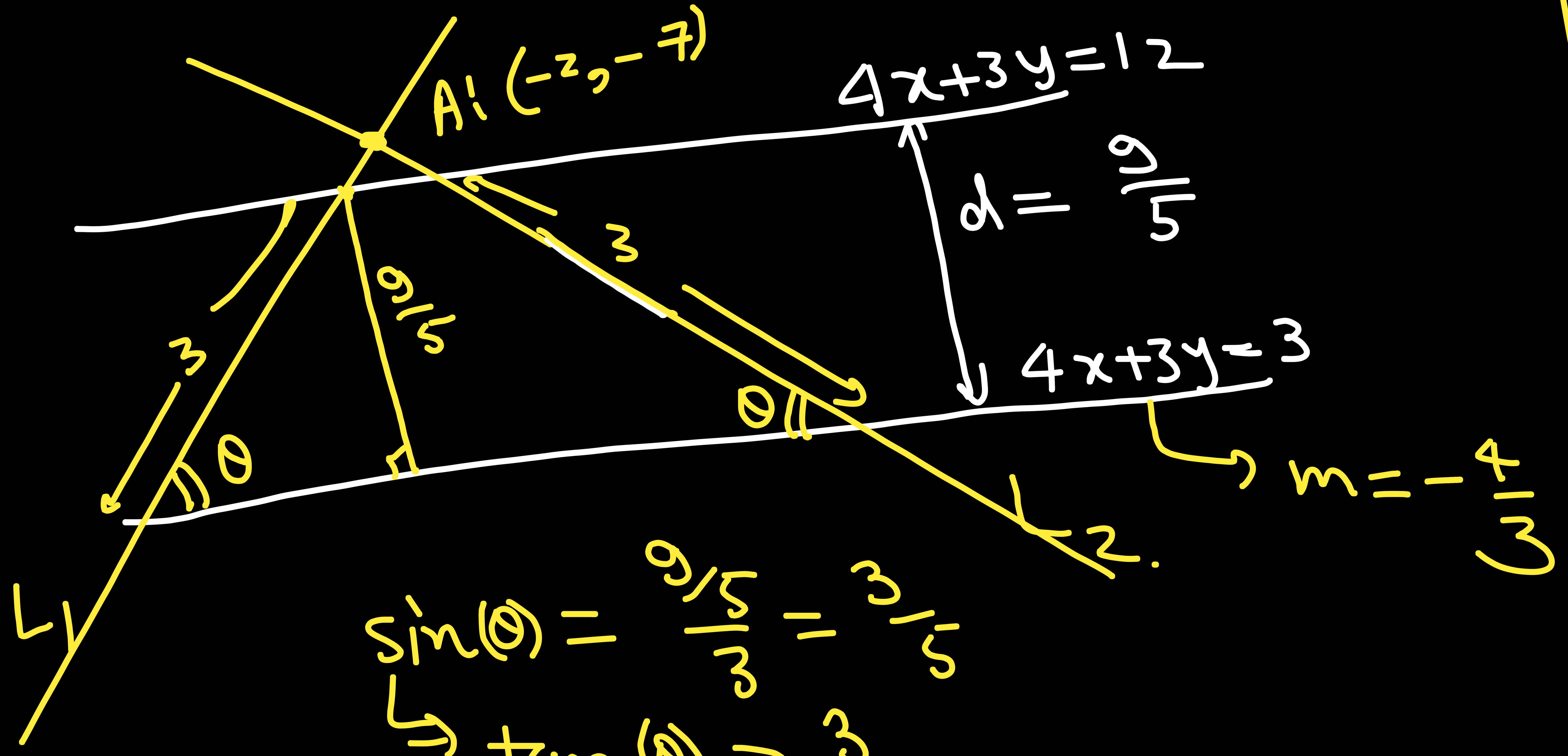
Clearly

Required $\dfrac{LR}{RM} = \dfrac{LR_0}{R_0M} = \dfrac{P'L}{QM} = \boxed{\dfrac{b}{d}}$ Ans

(as $\Delta MQR_0 \sim \Delta LP'R_0$)

Que

Find the equation of the straight lines passing through (-2, -7) and having intercept of length 3 units between the straight lines 4x + 3y = 12 and 4x+ 3y = 3.



$$\sin(\theta) = \frac{9/5}{3} = \frac{3}{5}$$

$$\hookrightarrow \tan(\theta) = \frac{3}{4}.$$

Let Required lines are L_1 & L_2

They will be Inclined at an $\angle$ $\tan^{-1}\left(\frac{3}{4}\right)$ with given lines

hence their slopes will be

$$\frac{m \pm \tan(\theta)}{1 \mp m\tan(\theta)} = \frac{-\frac{4}{3} \pm \frac{3}{4}}{1 \mp \left(-\frac{4}{3}\right)\left(\frac{3}{4}\right)}$$

$$= \frac{-\frac{4}{3} \pm \frac{3}{4}}{1 \pm 1}$$

$+ \rightarrow -\frac{7}{24}$

$\boxed{1 -} \rightarrow$ Undefined.

$$R.L_1 \equiv \boxed{y+7 = -\frac{7}{24}(x+2)}$$

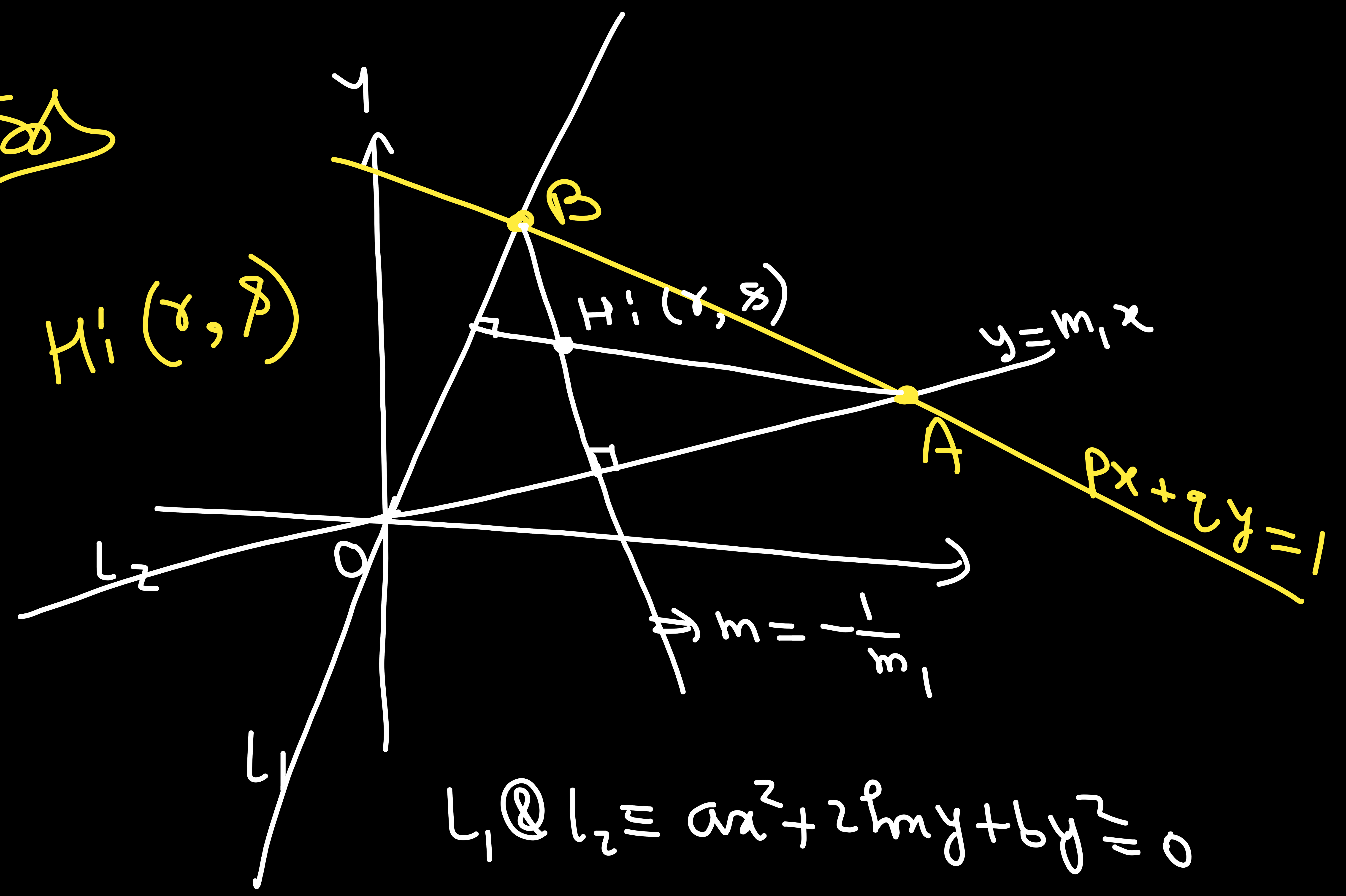
$(-2, -7)$

$$RL_2 \equiv \boxed{x+2=0}$$

Que

If orthocentre of the triangle formed by $ax^2 + 2hxy + by^2 = 0$ and $px + qy = 1$ is (r, s) then prove that $\frac{r}{p} = \frac{s}{q} = \frac{a+b}{aq^2 + bp^2 - 2hpq}$.

Sol



$H : (r, s)$

$L_1 \;\&\; L_2 \equiv ax^2 + 2hxy + by^2 = 0$

for A solve $y = m_1 x$

& $px + qy = 1$

$\hookrightarrow px + qm_1 x = 1$

$x = \frac{1}{p + qm_1}$

$A : \left(\frac{1}{p+qm_1}, \frac{m_1}{p+qm_1}\right)$

$|||^{ly}$ $B : \left(\frac{1}{p+qm_2}, \frac{m_2}{p+qm_2}\right)$

$$B \equiv \left(\frac{1}{p+qm_2}, \frac{m_2}{p+qm_2}\right)$$

$$BH \equiv y - \frac{m_2}{p+qm_2} = -\frac{1}{m_1}\left(x - \frac{1}{p+qm_2}\right)$$

$$AH \equiv y - \frac{m_1}{p+qm_1} = -\frac{1}{m_2}\left(x - \frac{1}{p+qm_1}\right)$$

$$\frac{m_1}{p+qm_1} - \frac{m_2}{p+qm_2} = \left(\frac{1}{m_2} - \frac{1}{m_1}\right)x - \left(\frac{1}{m_2(p+qm_1)} - \frac{1}{m_1(p+qm_2)}\right)$$

$$\frac{\cancel{(m_1-m_2)}p}{(p+qm_1)(p+qm_2)} = \left(\frac{\cancel{m_1-m_2}}{m_1m_2}\right)x - \left(\frac{\cancel{(m_1-m_2)}p}{m_1m_2(p+qm_1)(p+qm_2)}\right)$$

$$\Rightarrow p + \frac{p}{m_1m_2} = \frac{(p+qm_1)(p+qm_2)x}{m_1m_2}.$$

$$x = \frac{p(1+m_1m_2)}{(p+qm_1)(p+qm_2)}$$

$$\Rightarrow \frac{x}{p} = \frac{1+m_1m_2}{p^2+pq(m_1+m_2)+q^2m_1m_2}$$

$$ax^2+by^2+2hxy=0$$

$$m_1+m_2 = -\frac{2h}{b}$$

$$m_1m_2 = \frac{a}{b}$$

r

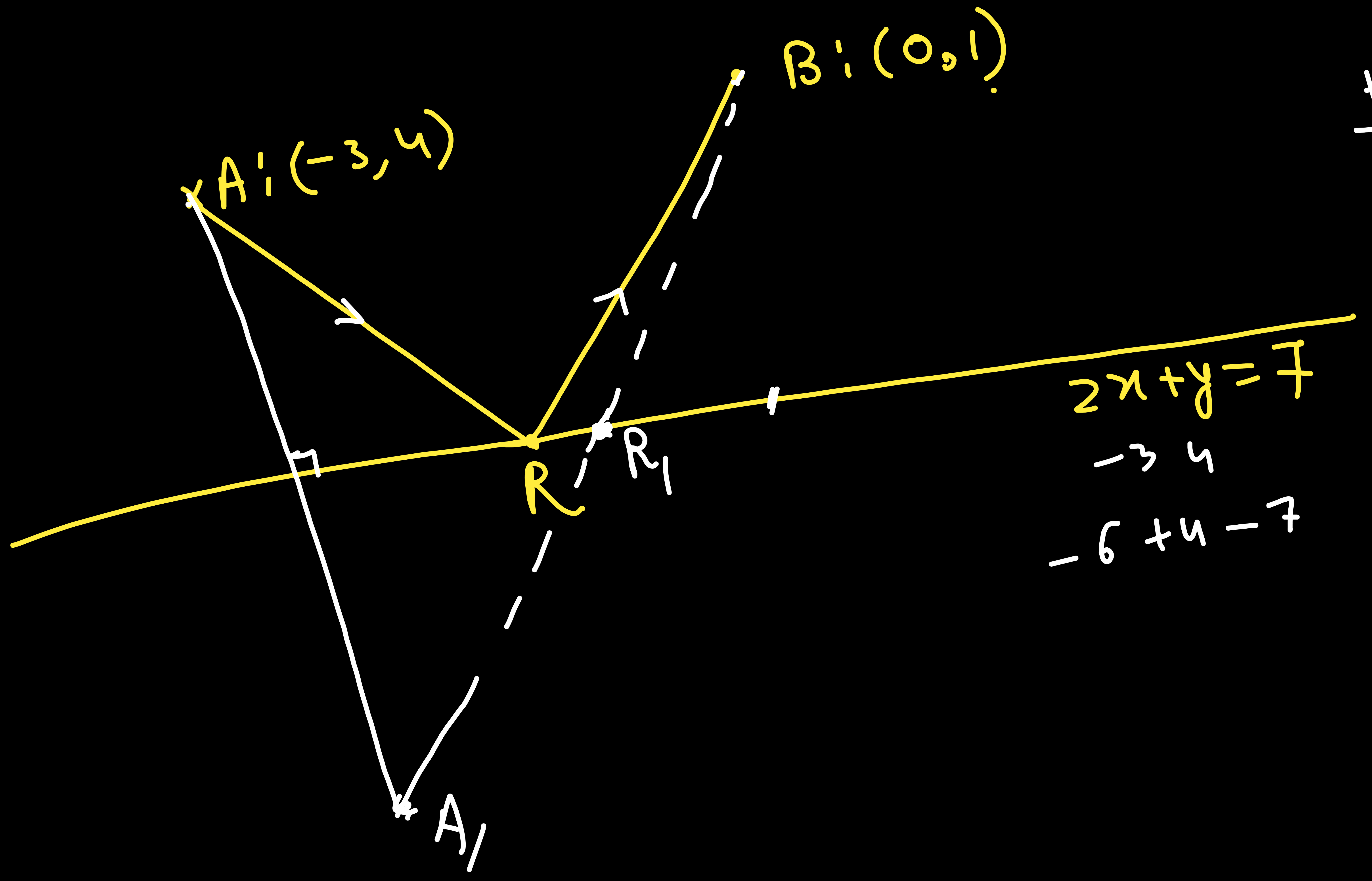
$$\frac{x}{p} = \frac{1+\frac{a}{b}}{p^2-\frac{2hpq}{b}+q^2\frac{a}{b}} = \frac{a+b}{p^2b-2hpq+aq^2}.$$

$$\frac{r}{p} = \frac{a+b}{bp^2-2hpq+aq^2}$$

Que

A ray of light generated from the source kept at (–3, 4) strikes the line 2x + y = 7 at R and then terminated at (0, 1). Find the point R so that ray travels through the shortest distance.

Sol



$-3 \quad 4$

$-6+4-7$

For A_1

$$\frac{x+3}{2} = \frac{y-4}{1} = \frac{-2(-9)}{5}$$

$\hookrightarrow A_1$